A Defense of Parity: Value Intransitivity Without a Money Pump

Choices are hard when one does not know how to compare items. Should you buy the milkshake that is on sale or the fruit smoothie that is healthier? For such choices, some claim one item must be better than the other unless they are precisely equally good (even if we cannot know which is true), while others claim the items are just not comparable. Still others claim the items’ values are comparable without being better, worse, or equally good because they relate by some fourth value relation. I am convinced by Ruth Chang’s (2002) argument that some items are comparable without being better, worse or equally good. However, Chang goes on to defend the existence of a relation she calls ‘parity’, and here I lend the argument more support. Martin Peterson (20017, 2014) uses money pumps to argue against all fourth value relations because they are all intransitive, but I claim money pumps are a problem with all fourth value relations except parity.

§II. The argument for parity

It is often assumed that value is like height in that something either has more, less, or the same amount of it. Chang calls this the Trichotomy Thesis (Chang 2002). She denies the Trichotomy Thesis using the small improvement argument, which depends on the existence of what I will call the small improvement phenomenon: Sometimes neither of two items is better than the other, and then one is slightly improved, yet still neither item is better. For instance, suppose you are deciding between two otherwise identical job offers, except that the tasks in job A are more exciting, whereas the work environment in job B is more pleasant. Let us also stipulate that neither offer is better nor worse than the other. Furthermore, the overall goodness of each offer is comparable—the different factors can each be weighed on the same choice-worthiness scale.
This is clear because a notable improvement of either factor would outweigh a nominal improvement of the other factor (see Chang’s 2002 “nominal-notable test”). If the Trichotomy Thesis were true, then these job offers could only be equally good. And if they were equally good, then the slightest improvement to either offer would tip the scales. Yet, this does not seem to be the case. For example, suppose a coffee mug is added to the second offer. It is still allowable to choose either job offer because a coffee mug does not tip the scales. So the offers are not equal; they relate in some other way.

In Mark Schroeder’s terminology, values are ‘weakly incomparable’ when one is neither better nor worse nor equally as good as the other.¹ Some purported versions of weak incomparability include vague equality (Broome 1997), crude or clumpy equality (Benbaji 2009), and parity (Chang 2002).² I will refer to all versions of weak incomparability other than strong incomparability as fourth value relations. Chang argues for parity by rejecting the other fourth value relations one-by-one. I buttress her argument by claiming only parity avoids problems with money pumps.

§III. Parity. What is it?

Parity is difficult to precisely define, but at least three traits are clear. First, items are on a par iff fully informed agents have a choice about which item to value more.³ This is the central trait of parity and can be used as a definition. Chang (2013) claims ‘given reasons’ constrain the roles of ‘voluntarist reasons’. Given reasons are not grounded in volitions; voluntarist reasons are

¹ See the Stanford Encyclopedia of Philosophy on ‘Value Theory’: <https://plato.stanford.edu/entries/value-theory/>
² An example of another candidate I will not consider is imprecise equality (Parfit 2016). But my argument generalizes to any other candidate.
³ When items are incomparable, one might be permitted to value either more (a counterexample to the ‘if’ part of the biconditional), but this is irrelevant if Chang’s more general argument against incomparability succeeds.
grounded in volitions. An informed agent can reasonably value drinking beer more than drinking coffee or vice versa based on voluntarist reasons, but given reasons constrain his evaluations such that he must value his family’s wellbeing more than drinking beer. Sports fans experience a paradigm case of parity when they commit to cheering for a team. A choice to become a Cubs fan can give one a voluntarist reason to value the Cubs winning more, even though the Yankees winning is objectively on a par.

Second, parity is a value relation in the same category as better, worse, and equally good. These are all relations between objects of assessment, not between mental states. An item is better, worse, equally good, or on a par with another item, whereas agents evaluate, prefer, believe, and so forth. Although there is no attitude of parity, an agent can believe that two items are on a par without personally evaluating—i.e., just float along without committing to either option if the possibility to choose ever arises. This may not itself be irrational, but it can be problematic practically. For instance, you might correctly believe that Al and Bill are on a par as romantic partners for you. Until you commit to pursue one of them, you have no more reason to pursue either of them, which leads to a standstill; you become Buridan’s ass on the Dating Game show. But once you commit to pursue Al, you now have more reason to pursue him than Bill because you (optionally, but appropriately) value a relationship with him more. A similar process might also occur regarding career paths, major purchases, and so forth.

And third, parity explains the small improvement phenomenon. That is, it explains why you may sometimes be permitted to prefer either option before as well as after a small improvement. If the small improvement argument supports parity, then it must be the case that items on a par at least sometimes exhibit the small improvement phenomenon.⁴

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⁴ The key details of my favored account of parity are similar to Gert (2004, 2015). There are limits on the strengths of one’s attitudes towards objects of assessment, and those limits are represented by evaluation intervals. When
§IV. The money pump problem

All fourth value relations are themselves intransitive precisely because the small improvement argument succeeds.\(^5\) \(A\) can be on a par with (vaguely equal to, crudely equal to, etc.) both \(B\) and \(B+\), even though \(B\) is not on a par with \(B+\). Because fourth value relations are intransitive, each of the trades in a money pump is rationally permissible. A money pump is a set of trades to which a person repeatedly agrees, resulting in a loss each time. A money pump problem arises if agreeing to a money pump is permissible, and thus it is permissible to agree to unlimited losses. The steps in a money pump are each permissible because the following are true of some set of items:

(A) It is rationally permissible to trade \(A\) for \(B\);

(B) It is rationally permissible to pay money to upgrade \(B\) to \(B+\)

(C) It is rationally permissible to trade \(B+\) for \(A\).

The money pumper repeats the trades in (A) through (C) until the money pumpee trades away the amount of money in (B) an unlimited number of times. The pumpee is not obligated to be pumped—just rationally permitted—but a decision theory fails if it permits money-pumping because it fails to optimize outcomes.\(^6\)

\(^5\) By ‘intransitive’, I mean any relation that is not necessarily transitive, which is any relation \(R\) for which \(aRb\) and \(bRc\) do not entail \(a Rc\). Thus, I use the term to refer to all nontransitive relations, not merely intransitive relations in the narrow sense of never being transitive. Sometimes \(A, B,\) and \(C\) are all on a par, so the relation is transitive in that instance, but parity itself is intransitive in the sense that this is not true for every set of items.

\(^6\) See Gustafsson (2010), who argues that preferences that merely permit money pumps (“non-forcing” money pumps) entail a money pump problem so long as what he calls the ‘dominance principle’ is true.
The theories likely have an escape out of a synchronic version of the money pump. They all may hold that (A)-(C) are rationally impermissible together as a synchronic set, since they reduce to an unequivocal loss. But as Peterson (2007, 2014) notes, it is harder for non-trichotomous theories to deny that (A)-(C) are permissible diachronically. If one is not permitted to make these trades when one has future vulnerabilities of being money pumped, then it seems likely that one is just never permitted to make any of these trades—perhaps that one is never permitted to make trades that are not upgrades. It is not terribly implausible for a decision theory to disallow any trade in which one does not receive an upgrade, especially if there are good theoretical reasons for the prohibition. But such a principle also prohibits changing one’s mind about a decision in cases where either option is permissibly preferred, and this is more bizarre. For instance, it would entail that the job seeker is free to choose job A or job B, but he is not free to change his mind to job A after already choosing job B (in the absence of new reasons) because this would not be an upgrade. I cannot accept this result, so I move on to find a different response to the money pump problem.7

§V. Solution to the money pump problem

Parity is studied in axiology, whereas a money pump is a problem for a decision theory. So to use money pumps to argue against fourth value relations, one must connect axiology to decision theory in the right kind of way. One must ultimately argue that the existence of intransitive value

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7 Chang has no published response to the money pump problem, but Peterson (2007) alludes to a suggestion from Chang via email that the money pump problem is avoidable if there are multiple types of permissibility. The only published quotation from that email correspondence is that the kind of permissibility “of choosing between items that are equally good is a different kind… from the ‘permissibility’ of choosing between items on a par.” It appears she is suggesting that choosing between equally good items is arbitrated by rational permissibility whereas choosing between items on a par is arbitrated by something like permissibility of parity. I see no way to develop this response.
relations entails that fully informed, rational agents may have intransitive preferences. This is what I deny; parity does not give permission to have intransitive preferences.

It will first help to get clearer about what I mean by ‘preference’ and ‘evaluation’. I simply use ‘preference’ to refer to a comparative evaluation. So to prefer A to B is to value A more than B. I use ‘evaluate’ (i.e., the verb form of ‘value’) to refer to the mental state that ought to set the aims of decision-making, given that the evaluations are appropriate. ( Appropriateness, as I will further explain below, has to do with coherence and fittingness.) Prioritizing one’s schedule is a paradigmatic example of evaluating—it sets one’s aims. To evaluate is not the same as to believe something has a certain value. For instance, you might value getting a purple marker for your child more than finishing your work, even though you believe that finishing your work is actually more valuable. That is, you prefer the world in which things are worse for you but better for your child, knowing that the resulting states of affairs are worse overall. I merely use this example to demonstrate that believing to be valuable and valuing are separable, but the action is also plausibly praiseworthy.

Axiology is related to decision theory because of how beliefs about objective value constrain appropriate evaluations. Generally, preferences must fit believed value relations. If one believes that an item is better than another item, then one ought to prefer it. If one believes that two items are equally good, then one ought to be indifferent. If one believes two items are on a par, then one is permitted to prefer either item or be indifferent.

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8 This is one of the standard uses of the term ‘prefer’. For instance, the SEP article on ‘Preferences’ (by Sven Ove Hansson and Till Grüne-Yanoff) defines a preference as a “subjective comparative evaluation”. But ‘preference’ can be used in ways that connote an essential affective component or a merely self-interested attitude, and I aim to disambiguate from those uses.

9 To be clear, this may be an instance of agent-relative value, not parity (if the preferences are not optional).

10 Examples of agent-relative value might be exceptions.

11 Except perhaps in rare cases of parity in which one is permitted to value one item more or equally but not to value the other item more (such as when evaluation intervals only touch, not overlap).
However, preferences are not rational merely because they fit believed value relations. Because of parity, there is not just one ideal set of preferences. In the earlier example, the job-seeker would be rational in valuing job B more than job A because the jobs are on a par, even though the opposite would also be rational. This creates the need for an additional requirement that one’s preferences are coherent. Even then, an agent makes rational choices only if she follows the advice of decision theory. In summary, good decision-making has three kinds of criteria. The first two regard forming appropriate preferences, and the third regards making decisions that are rational based on one’s preferences:

(I) One’s preferences must fit believed value relations.
(II) One’s preferences must be coherent.
(III) One’s choices must be rational based on one’s preferences.

Criterion (I) universally deems some choices irrational for any informed agent. Criterion (II) requires transitivity. Criterion (III) might be satisfied when one’s choices optimize outcomes based on one’s evaluations (given that there are no deontic constraints).

When an agent with stable preferences is money-pumped, either (II) or (III) is violated; she either has intransitive preferences, or her choices do not optimize her preferences. The latter possibility is a kind of irrationality that would be problematic on any account. The former is specially problematic for any fourth value relation account. If the Trichotomy Thesis were true, then any fully informed agent who satisfies (I) would thereby satisfy (II) because one’s evaluations would be transitive simply by fitting objective value relations. But if parity exists, then criterion (II) is important to eliminate the money pump problem.

12 Criterion (II) is actually useful for normal people regardless of whether the Trichotomy Thesis is true because incoherent preferences will still arise when they do not fit objective values.
Let us consider a specific case to see how an account with parity may avoid the money pump problem. Suppose a shirt is on a par with both $10 and $11. An informed, rational person may agree to trade the shirt for $10, and an informed, rational person may agree to trade $11 for the shirt. If these steps are repeated, one would lose a dollar an unlimited number of times. I argue that agreeing to the money pump would be irrational for any one individual, however, since the ‘informed, rational person’ in each case cannot be the same person. That person would have to violate criterion (II) by valuing $10 at least as much as the shirt, $11 more than $10, and the shirt at least as much as $11. Although each choice in the money pump is allowed, all three choices are never permissible for the same agent (with stable preferences) because a rational agent will never have the set of intransitive preferences that would justify the trades.

Wlodek Rabinowicz (2012) proposes a similar but more direct solution to the money pump problem for parity, given a key assumption. Rabinowicz assumes that there are many permissible preference orderings and that these orderings are explanatorily basic. There are no orderings that justify all the trades in a money pump because no orderings are intransitive. On this account, criterion (II) is unnecessary for the same reason as it is unnecessary if the Trichotomy Thesis were true—every permissible ordering is itself transitive, and one’s preferences must correspond to a permissible ordering. Hence, criterion (I) would satisfy criterion (II) for a fully informed agent. I find Rabinowicz’s account explanatorily backwards—items are rankable differently because they are on a par, not on a par because they are rankable differently. But permissible orderings are conceptually helpful, and the account is generally similar to what I propose.

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13 In 2008, Rabinowicz uses permissible preference orderings. In 2012, he shifts to permissible sets of degrees of favoring. The difference does not affect this discussion, so I will use the 2008 version for convenience.

14 Permissible orderings might play a similar theoretical role in axiology to possible worlds in ontology.
§VI. Responses to some objections

Some might object that a transitivity requirement for preferences is too demanding.\(^\text{15}\) Indeed, it is demanding, especially as one’s preferences become more complete, but that does not make it false. Suppose you are indifferent between $10 and the shirt. Then, assuming money matters, you must value $10.01 more than shirt. On any account, if you are indifferent between the shirt and both $10 and $10.01, then you can be money-pumped. When items are on a par, you may freely value either item more, but as soon as you set a point of indifference, the flexibility that parity originally gave you is gone. (Whether or not you are free to later change your evaluations is another matter.) This is demanding because it is easier to remain indifferent between the shirt and some range of prices; normal people should not be expected to have such precise preferences. I agree that money pumps are not dangerous enough to demand actual humans to bear the burden of precisifying preferences in all cases. But transitive preferences are still better, all else being equal. And as I will show below, other candidates for fourth value relations require intransitive preferences, which is a real problem.

One might also object that there is no good reason why evaluations could not themselves be on a par. I asserted above that this confuses categories because parity is an objective value relation, not a mental attitude. But there may be a coherent interpretation—perhaps valuing items on a par means that one is permitted to choose either item without changing one’s evaluations. The main problem with such an approach is that this obviously opens one up to money pumps because one’s choices will not be coherent.

\(^{\text{15}}\) This objection was raised by Michael Huemer in conversation.
An objection related to the previous one is that the precisification does not really represent one’s values if avoiding money pumps is the only goal; one ‘really’ values the items on a par.\textsuperscript{16} The strength of this argument depends largely on what it means to ‘really’ value something. Evaluations, if they are appropriate, are the mental states that ought to set the aims of decision-making. Precise evaluations aimed partly at avoiding money pumps should set the aims of one’s decision-making, so they are what one ‘really’ values. Hence, I do not feel the force of the objection.

Perhaps the objection is instead that only intrinsic attributes of items are reasons for evaluations, so avoiding money pumps is not ‘really’ a reason to value something. But it is just not true that we should base our evaluations solely on intrinsic attributes of things. A parent who appropriately prioritizes her child’s interests over her own may not do this because her child’s interests are intrinsically more valuable. A stamp collector who prioritizes acquiring stamps over coins does not do this because stamps are intrinsically more valuable. We have a variety of reasons for evaluations, and some of the reasons are merely voluntarist.

Most likely, all of these objections are ultimately motivated by the fact that we have many underdetermined and vague evaluations, and we are not blameworthy for having them. For instance, suppose the stamp collector values a rare stamp as much as $100. When pressed, he will probably say that he is not sure; he might only value it as much as $99. This is normal and excusable. It would clearly not be wise for normal humans to use their mental resources to completely precisify all of their evaluations. But it still is the case that, all else being equal, precise evaluations are better. To the extent that your evaluations are imprecise, you act wearing blindsers, without accurately knowing the targets at which you should be aiming. And my central

\textsuperscript{16} This objection was raised by Martin Peterson in email correspondence.
point is that precise evaluations help you avoid money pumps by promoting coherent decision-making.

§VII. Problem for other fourth value relations

While defining parity above, I noted that items are on a par exactly when volitions may rationally affect evaluations. This can be used as a definition, and I will do so now. So if a value relation permits a choice about what to prefer, then that value relation is parity. Fourth value relations need this flexibility to solve the money pump problem, so only parity can avoid the problem. If it were true that a specific preference fits each value relation, and value relations are intransitive, then preferences would have to be intransitive as well, and the money pump problem would remain.

For instance, let us see why crude equality permits intransitive preferences. I will use Yitzhak Benbaji’s (2009) account as a representative. Items are crudely equal by being in the same low-resolution set of items. The level of resolution is determined by the kinds of things compared. Benbaji claims strict equality only obtains between items that are equal in every contributing factor. In all other cases (which is most cases), they can only be crudely equal. Sometimes comparing only a subset of items raises the level of resolution. For instance, when $10 and $11 are compared in isolation, $11 is strictly better. Yet the shirt, $10, and $11 may be a crudely equal set. So $11 is better than $10 even though they are crudely equal in a certain context.

An account of crude equality can only solve the money pump problem if it requires transitive preferences. Instead, not only does it permit intransitive preferences, but it even requires them. We must first figure out which preference(s) fit crude equality to see whether
transitive preferences can be required by an account affirming crude equality. Since crudely equal items are crudely equal, I assume that one ought to be indifferent between them. If so, then money pumps remain a problem because the shirt is crudely equal to both $10 and $11, and trades are permissible when one is indifferent. Otherwise, if crude equality permits a range of evaluations, then it should be called ‘crude parity’. Crude parity permits transitive evaluations, but crude equality does not.

Similar problems arise with vague equality. Vagueness denies the Trichotomy Thesis by positing that some value comparisons have indeterminate borderline cases. Just as it might be indeterminate when a man passes from not bald to bald, it might be indeterminate whether or not $10 or $11 is better than the shirt. If one ought to be indifferent between all vaguely equal items, then money pumps are clearly a problem because it would be permissible to trade the shirt for $10 and then trade $11 for the same shirt. On the other hand, if vague equality offers freedom of preferences, then it is a version of parity. Perhaps this is what proponents of vague equality have in mind, but if so, then ‘vague equality’ is not an apt name—it is ‘vague parity’. Vague parity permits transitive evaluations, but vague equality does not.

The conclusion should not be too surprising: Fittingness requirements for evaluations must be flexible to require transitive preferences despite intransitive value relations. If a relation offers this flexibility, then (by definition) it is a version of parity. The existences of the candidates for fourth value relations are not mutually exclusive (at least not obviously), but if values are comparable without being better, worse, or equally good, then they must relate by some version of parity. Otherwise, intransitive relations will require intransitive preferences, and money pumps will be a problem.
I may need to be modest about the importance of this conclusion though. Money pumps are problems in decision theory, not axiology. It could be the case that there really are value relations that require intransitive preferences; so much the worse for us. In other words, the argument from money pumps is merely a prudential argument against other fourth value relations. The argument is only as strong as our reasons to expect that objective value relations would not doom our decision-making in this way. The prudential argument might be stronger if intransitive preferences are themselves irrational, since then the money pump problem would just be an additional problem. However, the argument would still be prudential; it would only be as strong as our reasons to expect value relations not to require irrationality.

§VIII. Summary

Parity is an objective value relation which permits volitional freedom of preference and exhibits the small improvement phenomenon. The existence of parity can be defended by rejecting the Trichotomy Thesis using the small improvement argument and then rejecting other fourth value relations because they require intransitive preferences. The latter step is my contribution. All fourth value relations are intransitive, so if preferences were inflexibly required to fit those relations, then they would also be intransitive. It is plausible that intransitive preferences are themselves irrational, and even if they are not, they result in a money pump problem. Out of all fourth value relations, only parity avoids the money pump problem because it offers the flexibility to require transitive preferences. This is merely a prudential argument against other fourth value relations, but I doubt that value relations would doom decision-making in this way.
Works Cited


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